1. Give standard definitions for the following concepts:
   a. valid arguments. (8 points):
   b. truth-functional connectives. (7 points)

2. Decide the truth value of each of the following statements. (4 points each)
   (1) If every sentence of an argument is false, then the argument must be invalid.
   (2) An inconsistent set of sentences logically implies every sentence.
   (3) Every tautology is logically equivalent to every tautology.
   (4) The set \( S = \{ \neg P, P \supset Q, Q \supset R, \neg Q \supset R \} \) is a consistent set.
   (5) “\( (\exists x)(\exists y)(Fx \& Fy) \)” logically implies that at least two different things are \( F \).

3. Multiple choices: (Write down “a” or “b” as your answer to each question. 4 points for each)
   (1) ( ) “\( P \equiv \neg P \)” (a: is; b: isn’t) a logical contradiction.
   (2) ( ) “\( P \)” is (a: logically equivalent; b: not logically equivalent) to “\( P \lor (P \& Q) \)”.
   (3) ( ) “\( (x)((Ax \& Bx) \lor \neg Cx) \)” is (a: logically equivalent; b: not logically equivalent) to “\( \neg (\exists z)(\neg (A\neg z \lor B\neg z) \lor (\neg C\neg z \land \neg C\neg z)) \)”.
   (4) ( ) “\( T \)” (a: follows; b: doesn’t follow) logically from \( \{ P, P \supset Q, Q \supset R, \neg S \supset \neg R, \neg S \lor T \} \).
   (5) ( ) a: “Haa” logically implies “\( (\exists y)(\exists z)Hyz \)”;
        b: “Hab” logically implies “\( (\exists y)(\exists z)(y \neq z \& Hyz) \)”.

4. Prove (formalize them first when required) that the following arguments are VALID, using whatever deductive method you know. (10 points each.)
   a. At least one man loves every dog. Therefore, every dog is loved by at least one man. (Use the following key for translation. \( Mx \): x is a man; \( Dx \): x is a dog; \( Lxy \), x loves y.)
   b. At least three things are heavy. Therefore, at least two things are heavy. (Use the following key for translation. \( Hx \): x is heavy.)
   c. 1. \( P \supset Q \)
      2. \( \neg P \supset R \)
      3. \( \neg Q \supset \neg R \) \hspace{1cm} \therefore Q

5. Show (formalize them first when required) by whatever semantic method you know that the following arguments are INVALID. (5 points for a; 10 points for b.)
   a. Every dog is loved by at least one man. Therefore, at least one man loves every dog. (Use the following key for translation. \( Mx \): x is a man; \( Dx \): x is a dog; \( Lxy \), x loves y.)
   b. \( P \supset ((Q \& R) \supset S) \hspace{0.5cm} \therefore (P \supset (Q \& R)) \supset S \)